Schroer:

1) Construction of "$\frac{1}{N}$ spinors"

2) Chiral Aharonov-Bohm construction of Ising order disorder variables

3) $\langle\psi\rangle \neq 0$ in the Schwinger model

Starting with boson-free theory
by putting two sources $\left\{\begin{matrix}\text{electric}\\ \text{magnetic}\end{matrix}\right\}$ sources

Mandelstam bosonization of γ.
Kadanoff idea

$\det(i\hat{D\!\!\!/})$ $\qquad\qquad i\partial\!\!\!/ = i\partial\!\!\!/_+ + i\partial_-$

$$\| \qquad \int \mathcal{D}\psi\mathcal{D}\bar{\chi}\, e^{-i\int \bar{\chi}\cdot\hat{D}\psi} = e^{-\Gamma}$$

$$\Gamma = \mathrm{Re}\,\Gamma + i\underbrace{\mathrm{Im}\,\Gamma}$$

$\downarrow \zeta'(0)$ $\qquad$ related the 3diml Dirac op. $\eta(0)$

anomaly

$$\partial^\mu j^W_\mu = c\, d\Lambda$$

$$\Gamma = \frac{1}{4\pi}\iint F(x)\,D(x-y)\,F(y) + \frac{i}{4\pi}\int F(x)\,D(x-y)\,\partial^\mu A_\mu$$

$$A^{AB}_\mu = 2\pi\int_P^\infty \varepsilon_{\mu\nu}\,\delta(x-\xi(t))\,d\xi_\nu$$

massive Majorana spinor field is useful to describe Ising model.

Since 1926 physics based on 2 ingredients

- quantum mech.
- general rel.

Inconsistent

Point particle

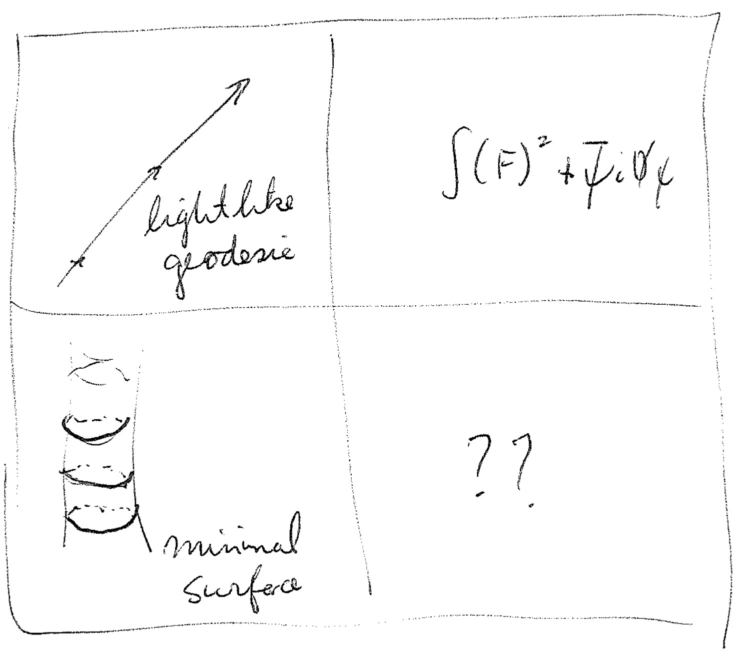
$$\int d\tau \; g\left[\tfrac{1}{2} g_{ij} \frac{dx^i}{d\tau}\frac{dx^j}{d\tau} + \tfrac{1}{2}\bar{\psi}_i \frac{D}{D\tau}\psi^i\right]$$

After quantization get massless particles of spin $1, \frac{1}{2}$

$\longrightarrow$ linear approx $\left\{\begin{array}{l}\text{Maxwell} \\ + \text{Dirac}\end{array}\right.$ equations

Do they have a natural non linear generalization?

Yes supersymm YM.



Action for min. surf $\quad I = \int d^2\sigma \cdot \left[\begin{array}{l}\text{area per} \\ \text{unit } \sigma\end{array} + \underbrace{\bar{\psi} \cdots \psi}_{\text{supersymmetric gen.}}\right]$

Quantization $\Rightarrow$ unitary Poincaré action on Hilb. space

Many coincidences occur in 10 dim's - need for supersymmetric model

What is the non-linear theory related to minimal surfaces the same way supersymmetric YM is related to geodesics.

~~Transfer operators~~

Supersymmetric theory.

QM on M $\xrightarrow[\text{integral}]{\text{path}}$ integration over LM

SQM on M $\longrightarrow$ diffl forms over LM | non-linear σ-model (0+1-dimensions)

But now suppose we try to do quantum mechanics over LM, i.e. we do string theory.

QM on LM $\longrightarrow$ (integration on maps of R.S.'s into M.)

Let us start with a review of the Hall effect model. I want to consider a bundle of Hilbert spaces over the 2-torus T, with a family of Hamiltonians. Now this won't lead to an index problem, but maybe it is possible to make a fermion version which would be interesting ~~I have a feeling it ought to be trivial.~~ The parameter B is missing. Also the Hamiltonian is quadratic in the fluxes

Mathematically what do we have? Bundle of Hilbert spaces over T with filtration defined by H. Generic

Bohm-Aharonov effect. Here the point is that when configuration space is not simply-connected, one gets various quantizations resulting from the fact that one can tensor with flat line bundles. Put another way ~~one can the path integral is taken over~~ the path space splits into components, and one can modify the action by a character of the $\pi_1(M)$, that is, by a flat line bundle over M.

So now let us return to the situation of the Hall effect. Configuration space for a single particle has the homotopy type ∞ and hence we have a 2-parameter family of line bundles. I think we have the same underlying Laplacean-type Hamiltonian being twisted by this family of line bundles.

It should be possible to see ~~these line bundles~~ a simpler model of this phenomena. So take the situation of a Laplacean on a manifold with a 2-pararameter family of flat line bundles. Now you want to understand the spectrum or maybe just the ground state variation.

Problems to look at: Can examine the

Mathematical translation. One works over a non simply-connected manifold M with a Laplacean or Dirac type operator. Then one twists with respect to flat line bundles on M, getting a family of Hamiltonians.

~~[illegible]~~ First question is how to think of the Hilbert bundle. Does the whole story come apart over the universal covering?

Up on $\tilde{M}$ I can trivialize ~~[illegible]~~ somehow. Somehow the ~~spectrum of~~ the operator does not change ~~upon~~ upstairs