Getzler – (Raoul's office) 12/21/83

$$H = \frac{1}{2}\left[-\Delta + (Ax, Ax)\right] = \frac{1}{2}\left[-\Delta + A^*A\right]$$

A antisymmetric matrix on $\mathbb{R}^{2n}$

He works out the heat kernel of H as a power series in A. Later A is to become a matrix of 2-forms, essentially the curvature.

$$k_t(x,y) = \frac{1}{(2\pi t)^n} \underbrace{\hat{A}(tA)}_{\text{pfaffian}\left(\frac{tA}{\sinh tA}\right)} \exp\left[-\frac{1}{2t} \left(\cosh(tA)(x^2+y^2) - 2xy\right)\right]$$

$$\frac{tA}{\sinh(tA)}$$

Dirac in normal coords or synchronous frame

$$g_{ij} = \delta_{ij} + O(|x|^2)$$

$$\Gamma^k_{ij} = O(|x|)$$

connection in normal coords using the synchronous frame

$$A^a_{ib} = \frac{1}{2} R_{ij}{}^a{}_b\, x^j + O(|x|^2)$$

$$\not{D}^2 = g_{ij}\left(\partial_i + \tfrac{1}{2}A_i\right)\left(\partial_j + \tfrac{1}{2}A_j\right) + \Gamma^k_{ij}\left(\partial_k + \tfrac{1}{2}A_k\right) + \frac{R}{4}$$

first 2 terms are just the trace Lap.

$\frac{1}{2}$ because it acts on spinors

Both: E_i horizontal vector fields

$$[E_i, E_j] = R_{ij}{}^a{}_b\, X^b_a$$

$$\not{D} = e_i E_i \qquad \square = -E_\alpha E_\alpha + \frac{R}{4}$$

leading part is $e^a e^b$

$$D_0 = \sum_{i=1}^{2n} \left(\partial_i + \frac{1}{4} R_{ij}{}^a{}_b \, x^j\right)^2$$

Def: $k_t(x,y) \in C^\infty\left((0,1) \times \mathbb{R}^{2n} \times \mathbb{R}^{2n}\right)$ is in the class $C(m)$ if

$$\left|\partial_x^\alpha k_t(x,y)\right| \leq \text{const } t^{-\frac{n}{2} - \frac{|\alpha|}{2}}\left(t^{\frac{m}{2}} + |x|^m\right) e^{-O\left(\frac{|x-y|^2}{t}\right)}$$

? n maybe

D_0 still not the final result because it uses the Clifford multiplication. We want

$$D_1 = \sum_{i=1} \left(\partial_i + \frac{1}{4} R_{ij} \, x^j \wedge\right)^2$$

wedge on forms.

Here $R_{ij} = R_{ijkl} \, dx^k dx^l$

He thinks of e^a as interior + exterior and the interior product as a lower order term.

$\mathrm{Cliff}(\mathbb{R}^{2n})$ is an $SO(2n)$-module
$\cong$
$\Lambda(\mathbb{R}^{2n})$ ——— "

These are isomorphism of these two modules which he calls a symbol map. Now there is an analogue of the formula for the composition of ΨDO's.

1) $\sigma(a \circ b) = \sigma(a) \wedge \sigma(b)$ + lower order
2) Projection onto $e_1 \cdots e_{2n}$ is proportional to $\mathrm{tr}_{\Delta^+} - \mathrm{tr}_{\Delta^-}$

$$k_t(x,y) \in \mathrm{Hom}(\Delta_y, \Delta_x)$$
$$\underset{\text{framing}}{\cong} \quad \mathrm{End}(\Delta) = \mathrm{Cliff}(\mathbb{R}^{2n})$$

Let $k_t(x,y)_m$ be the piece of k_t lying in $\Lambda^m \oplus \cdots \oplus \Lambda^{2n}$ $\quad 0 \le m \le 2n$. $\quad$ Facts

Assertion:
a) $k^1_t(x,y)_m$ is in $C(m)$ $\qquad$ can be 2.
b) $[k_t(x,y) - k'_t(x,y)]_m$ is in $C(m+1)$

Proof by induction on m. starting from McKean Singer for $m = 0$. Then b) for $m = 2n$
$$\Rightarrow \mathrm{tr}_{\gamma_5}\left(k_t(x,x) - k'_t(x,x)\right) \longrightarrow 0 \qquad \text{as } t \to 0$$

To prove a), b) we use

i) $\quad x^\alpha \partial^\beta_x k_t(x,y) \in C(m + |\alpha| - |\beta|)$ if $k_t \in C(m)$

ii) $\quad k^i_t(x,y) \in C(m_i) \quad i = 1, 2$

$$\int_0^t \left[\int k'_s(x,z)\, k^2_{t-s}(z,y)\, dz\right] ds \in C(m_1 + m_2 + 2)$$

(think $\displaystyle\int_0^t e^{-sA} e^{-(t-s)B}\, ds$)

iii) (Duhamel's formula) If k^i_t is the heat kernel of A^i, then

$$k^1_t - k^2_t = \int_0^t \left\{\int k_s(x,z)\, (A^1 - A^2)_z\, k_{t-s}(z,y)\, dz\right\} ds$$

$$e^{-tA} - e^{-tB} = \int_0^t ds\; e^{-sA}(A-B)e^{-(t-s)B}$$

$$\not{D}^2 = \underset{\substack{\downarrow \\ \delta_{ij}}}{g^{ij}} \underset{\substack{\downarrow \\ \partial_i + \frac{1}{4} R_{ij} \wedge}}{\left(\partial_i + \tfrac{1}{2} A_i\right)} \left(\partial_j + \tfrac{1}{2} A_j\right) + \underset{\substack{\downarrow \\ 0}}{\Gamma^k_{ij}} \left(\partial_k + \tfrac{1}{2} A_k\right) + \underset{\substack{\downarrow \\ 0}}{\frac{R}{4}}$$

because $A^a_{ib} = \frac{1}{2} R^a_{ij\,b}\, x^j + O(|x|^2)$

This is the crux of the approximation Luis makes in the path integral.