T I guess I'm trying to understand the bigraded diffl alg $T\left((\tilde{A}\oplus\mathbb{C}\varepsilon/\mathbb{C})^*\right)$ as $\mathbb{C}\langle\theta^i, \alpha, ?\rangle$

$? = \nabla\alpha = d\alpha + [\rho,\alpha]$

No the important thing maybe about

$d\alpha$

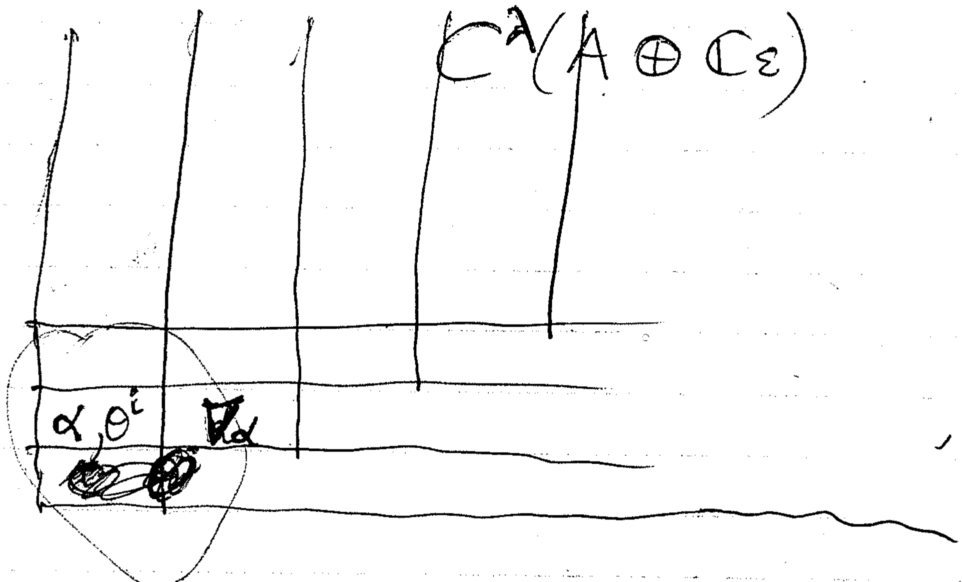
$$0 = d'\chi + \chi^2 = d'\alpha + d'\rho + \alpha^2 + [\alpha,\rho] + \rho^2$$
$$= d'\alpha + [\alpha,\rho] + \alpha^2 + \omega$$

is that $d'\alpha \in \mathbb{C}\langle \rho, \theta^i, \alpha\rangle$ so that we can also take $d''\alpha = \varphi$ as our last element. So anyway ~~[illegible]~~.

$$\mathbb{C}\langle\theta^i\rangle * \mathbb{C}\langle\alpha, d''\alpha\rangle$$

closed under $d' + ad_\rho$

$C^\lambda(A\oplus\mathbb{C}\varepsilon)$



Note that
$$\varphi = d\chi + \chi^2 = d(\rho+\alpha) + (\rho+\alpha)^2$$
$$= \underbrace{d\alpha + [\rho,\alpha]}_{\nabla\alpha} + \alpha^2 + \omega$$

Shows that φ not homogeneous for the D-grading

U Try again.

$$T(\bar{A}^*) \subset T\big((\overline{\widetilde{A}\oplus\mathbb{C}\varepsilon})^*\big) \xrightarrow{\sim} T\big((A\oplus\mathbb{C}\varepsilon)^*\big)$$

$$T\big((\overline{\widetilde{A}\oplus\mathbb{C}\varepsilon})^*\big) \subset T\big((\widetilde{A}\oplus\mathbb{C}\varepsilon)^*\big) \longrightarrow T(A\oplus\mathbb{C}\varepsilon)^*$$

This composition is an isomorphism of algebras probably compatible with the bigrading, the vertical differential, but not the horizontal diffl as $d(\varepsilon) = c^1 \notin A$

The idea is that $T\big((\widetilde{A}\oplus\mathbb{C}\varepsilon)^*\big)$ should be a bigraded algebra with derivation $\nabla = \nabla' + \nabla''$ where ∇', ∇'' are diffls but $[\nabla', \nabla''] = \operatorname{ad}\omega$

So far we have calculated

$$T\big((\overline{\widetilde{A}\oplus\mathbb{C}\varepsilon})^*\big) = \mathbb{C}\langle \theta^i, \alpha, \nabla\alpha \rangle$$

where
$$\nabla\theta^i + f^i_{jk}\,\theta^j\theta^k = 0$$
$$\nabla(\alpha) = \nabla\alpha = \varphi - \alpha^2 - \omega$$
$$\nabla(\nabla\alpha) = [\omega, \alpha].$$

Calculate the above composition

not compatible with d.

$$\begin{array}{ccccc}
\mathbb{C}\langle \theta^i, \alpha, \nabla\alpha\rangle & \subset & \mathbb{C}\langle \rho, \theta^i, \chi, \varphi\rangle & \twoheadrightarrow & \mathbb{C}\langle \rho, \theta^i, \varphi\rangle \\
\theta^i & \longmapsto & & \longmapsto & \theta^i \\
\alpha & \longmapsto & \chi - \rho & \longmapsto & -\rho \\
\nabla\alpha & \longmapsto & \varphi - \alpha^2 - \omega & \longmapsto & \varphi - \rho^2 - \omega
\end{array}$$

V $\quad \mathbb{C}\langle \theta^i, \alpha, \nabla\alpha \rangle \xrightarrow{\sim} \mathbb{C}\langle \rho, \theta^i; \varphi \rangle$

$$\theta^i \longmapsto \theta^i$$
$$\alpha \longmapsto -\rho$$
$$\nabla\alpha \longmapsto \varphi - \rho^2 - \omega$$

I want to calculate ∇ on the right

$\nabla\theta^i + f^i_{jk}\,\theta^j\theta^k$ on the left

but $\quad d'\theta^i + [\rho, \theta^i] + f^i_{jk}\,\theta^j\theta^k = 0 \quad$ on the right

$\quad d'\rho + \rho^2 + \underbrace{f^0_{jk}\,\theta^j\theta^k}_{-\omega} = 0 \quad$ on the right

$$\begin{array}{ccc} \nabla\alpha & \longmapsto & \varphi - \rho^2 - \omega \\ & & \varphi - \rho^2 + f^0_{jk}\,\theta^j\theta^k \\ \nabla \uparrow & & \\ \alpha & \longmapsto & -\rho . \end{array}$$

$$\begin{array}{ccc} \theta^i & \longmapsto & \theta^i \\ \nabla \downarrow & & \downarrow \text{“}\nabla\text{''} \\ -f^i_{jk}\,\theta^j\theta^k & \longmapsto & -f^i_{jk}\,\theta^j\theta^k \end{array}$$

$\therefore$ “∇''$\theta^i = (d' + \mathrm{ad}\,\rho)\,\theta^i$

“∇''$\rho = -\varphi + \rho^2 + \omega$

“∇''$\rho = (d' + \mathrm{ad}\,\rho)\,\rho - \varphi$

$$(d' + \mathrm{ad}\,\rho)(\rho) = d'\rho + 2\rho^2$$
$$= d'\rho + \rho^2 + \rho^2$$
$$= \omega + \rho^2$$

W $\quad \mathbb{C}\langle \theta^i, \alpha, \nabla\alpha \rangle \longrightarrow \mathbb{C}\langle \rho, \theta^i, \varphi \rangle$

$$\| \qquad\qquad\qquad \|$$

$$T\left(\left(\tilde{A} \oplus \mathbb{C}\varepsilon/\mathbb{C}\right)^*\right) \qquad T\left(\left(A \oplus \mathbb{C}\varepsilon\right)^*\right)$$

$$\left(\begin{array}{l} \nabla\theta^i + f^i_{jk}\theta^j\theta^k = 0 \\ \nabla(\alpha) = \nabla\alpha \\ \nabla(\nabla\alpha) = [\omega, \alpha] \end{array}\right. \qquad \left(\begin{array}{l} d'\rho + \rho^2 = \omega \\ (d' + \mathrm{ad}\,\rho)(\theta^i) + f^i_{jk}\theta^j\theta^k = 0 \\ d'\varphi = 0 \end{array}\right.$$

(because $d'\varphi + [\chi, \varphi] = 0$)
in $T\left(\left(\tilde{A} + \mathbb{C}\varepsilon\right)^*\right)$ and $\chi \mapsto 0$.

$$\theta^i \longmapsto \theta^i$$
$$\alpha \longmapsto -\rho$$
$$\nabla\alpha \longmapsto \varphi - \rho^2 - \omega$$

$$\theta^i \longleftarrow\!\shortmid \theta^i$$
$$-\alpha \longleftarrow\!\shortmid \rho$$
$$\nabla\alpha + \alpha^2 + \omega \longleftarrow\!\shortmid \varphi$$

Calculate $\nabla\rho$: $\quad \rho \longmapsto -\alpha \overset{\nabla}{\longmapsto} -\nabla\alpha \longmapsto -\varphi + \rho^2 + \omega$

$$\boxed{\nabla\rho = -\varphi + \rho^2 + \omega}$$

$$\varphi \longmapsto \nabla\alpha + \alpha^2 + \omega \overset{\nabla}{\longmapsto} \underbrace{[\omega, \alpha]}_{} + \overset{[\nabla\alpha, \alpha]}{(\nabla\alpha\,\alpha - \alpha\nabla\alpha)}$$

$$\longmapsto \cancel{[\omega, -\rho]} + [\varphi - \cancel{\rho^2} - \cancel{\omega}, -\rho]$$

$$\boxed{\nabla\varphi = [\rho, \varphi]}$$

X Check

$$\nabla(\nabla\varphi) = \nabla[\rho,\varphi]$$
$$= [\nabla\rho,\varphi] - [\rho,\nabla(\varphi)]$$
$$= [-\cancel{\varphi}+\rho^2+\omega,\varphi] - [\rho,[\rho,\varphi]]$$
$$= [\omega,\varphi] + \cancel{[\rho^2,\varphi]} - \cancel{[\rho^2,\varphi]}$$

$\nabla = (d' + \mathrm{ad}\,\rho)$ on θ^i

$$(d'+\mathrm{ad}\,\rho)(\rho) = d'\rho + [\rho,\rho]$$
$$= \omega + \rho^2 \quad = \nabla\rho + \varphi$$

$$(d'+\mathrm{ad}\,\rho)(\varphi) = d'\varphi + [\rho,\varphi] = [\rho,\varphi]$$

$\therefore\ \nabla = \boxed{d'+\mathrm{ad}\,\rho}$ on θ^i

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wait. You've calculated ∇ on $\mathbb{C}\langle\rho,\theta,\varphi\rangle$

You would like to split into horizontal and vertical components.

ρ, θ^i bidegree $(0,1)$
φ —— $(1,1)$

$d'+\mathrm{ad}\,\rho$ has bidegree $(0,1)$

$$\nabla' = d'+\mathrm{ad}\,\rho \qquad \nabla''\begin{pmatrix}\theta^i\\ \varphi\\ \rho\end{pmatrix} = \begin{pmatrix}0\\0\\-\varphi\end{pmatrix}$$

Y

$$[\nabla', \nabla'']\theta^i = (\cancel{\nabla'\nabla''} + \nabla''\nabla')\theta^i$$
$$= \nabla'' f^i_{jk}\theta^j\theta^k = 0.$$

$$\cancel{\nabla'\nabla''}\varphi + \nabla''\nabla'\varphi$$
$$= \nabla''([\rho, \varphi])$$
$$= [\nabla''\rho, \varphi] - [\rho, \cancel{\nabla''\varphi}]$$
$$= [-\varphi, \varphi] = 0$$

$$\nabla'\nabla''\rho + \nabla''\nabla'\rho = \nabla'(-\varphi) + \nabla''(\cancel{\varphi} + \rho^2)$$
$$= -[\rho, \varphi] + (\nabla''\rho)\rho - \rho(\nabla''\rho)$$
$$= -[\rho, \varphi] + -\varphi\rho + \rho\varphi = 0$$

So where are we now ??

We've gone back to

$$\begin{tikzcd}
C^\lambda(A) \arrow[d] & C^\lambda(\tilde{A} \oplus \mathbb{C}\varepsilon) \arrow[l, two heads] \arrow[d] & C^\lambda(A \oplus \mathbb{C}\varepsilon) \arrow[l] \arrow[dl, "\cong"] \\
\bar{C}^\lambda(A) & \bar{C}^\lambda(\tilde{A} \oplus \mathbb{C}\varepsilon) \arrow[l, two heads] &
\end{tikzcd}$$

$$\begin{tikzcd}
T(A) \arrow[d] & T(\tilde{A} \oplus \mathbb{C}\varepsilon) \arrow[l, two heads] \arrow[d] & T(A \oplus \mathbb{C}\varepsilon) \arrow[l] \\
T(\bar{A}) & T(\tilde{A} \oplus \mathbb{C}\varepsilon / \mathbb{C}) \arrow[l, two heads] &
\end{tikzcd}$$

question. In general go back to

$$\Omega \otimes W = \Omega_{hor} \otimes \Lambda \mathfrak{g}^*_A \otimes \Lambda \mathfrak{g}^*_\alpha \otimes S \mathfrak{g}^*_{\nabla\alpha}$$

$$\nabla = d - A^a L_a \quad \text{in general}$$

~~A~~ for some elements

$$\nabla x = (d + ad(A))(x)$$

e.g. α, $\nabla\alpha$.

$$\nabla A = dA + A^a[X_a, A]$$

$$= dA + [A, A] = (d + ad A)(A).$$

Could ask already for

$$\Omega = \Omega_{hor} \otimes \Lambda \mathfrak{g}^*_A$$

$$\nabla = d - A^a L_a$$

$$[\nabla, \iota_X] = L_X - [A^a L_a, \iota_X]$$

$$= L_X - \underbrace{[A^a, \iota_X]}_{\iota_X A^a} \underbrace{L_a}_{L_{X_a}} - A^a \iota_{[X_a, X]}$$

$$X = \iota_X(A^a) X_a \qquad \iota_X A^a \quad L_{X_a} \quad L_X$$

$$[\nabla, \iota_X] = -A^a \iota_{[X_a, X]}$$

~~A~~ Compare ∇ to $d + ad A$

$$\nabla A = dA + A^a[X_a, A] = dA + [A, A)$$

$$= \cancel{dA + A^2} \quad dA + A^2 + A^2$$

$$= (d + ad A) A.$$