CMI Summerschool 2012 / Course Schicho
Exercises Part 1

Exercise 1. In the game FlipIt, nodes of an undirected graph are labelled white or black. When you click a node, then the node and all adjacent nodes change color. Initially, every node is white. The goal is to make every node black.
Prove Top's Theorem: the above game has a solution for every finite graph.
Reference: `http://www.math.rug.nl/~top/perio.pdf` (in Dutch)

Exercises 2,3. In the game Salmagundy Solitaire, one starts with a tree with nodes labeled by integers. In each move, you make one of four possible moves:

1. replace #(a) by #(a-1)-(-1)
2. replace #(a)-(-1) by #(a+1) (reverse of 1)
3. replace #(a)-(b)## by #(a-1)-(-1)-(b-1)##
4. replace #(a)-(-1)-(b)## by #(a+1)-(b+1)## (reverse of (3)

Here, a,b represent arbitrary integers, and #,# represent arbitrary subtrees.
The goal is to produce the single node tree (1).
For the two trees below, one has a solution and one does not have. Find the solution for one and prove non-existence for the other.

```
(-2)-(-2)-(-2)-(-2)-(-1)-(-1)
            |
          (-1)

(-2)-(-2)-(-2)-(-2)-(-1)
            |
          (-1)
```

References: J. A. Morrow, Minimal normal compactifications of $\mathbb{C}^2$, Rice Univ. Studies, 1973, 97–112.
C. P. Ramanujam, A topological caharcterization of the affine plane as an algebraic variety, Ann. Math. 94, 1971, 69–88.