

CMI Summerschool 2012 / Course Schicho

Rules for Salmagundy

Quests. At any moment, the current situation in every quest is described as a simplicial complex Ξ , a dimension $d \in \mathbb{Z}_{>0}$, a generating degree $b \in \mathbb{Z}_{>0}$, an order $o \in \frac{1}{b}\mathbb{Z}_{\geq 0} \cup \{\infty\}$, and a monomial factor $a : \text{Vertices}(\Xi) \rightarrow \frac{1}{b}\mathbb{Z}_{\geq 0}$.

Dido's moves. If there is a quest and a face of Ξ with sum of labels greater than or equal one, then Dido may issue a blowup type I on that face.

If there is a quest with order ∞ , then Dido may issue a blowup type II on that quest.

If there is a quest with $o = 1$ and such that Ξ consists only of the empty (-1-)face, then Dido may issue a descent on that quest.

If there is a quest with $1 < o < \infty$ and such that Ξ consists only of the empty (-1-)face, then Dido may issue a tightification on that quest.

For a quest and a vertex $j \in \text{Vertices}(\Xi)$, Dido may issue an intersection quest.

Mephisto's moves. After a blowup, Mephisto decides which of the quests are not lost, and then he transforms all remaining quests and provides the new data to Dido. After a descent, tightification, or intersection, he provides the data for the new quest.

Restrictions for Mephisto. The quests which are higher in hierarchy than the quest on which the blowup is done must be kept (in particular the main quest can never be lost).

For any quest, d and b remain constant, and a keeps its values on existing vertices. The value of a on the new vertex is in the interval $[a(i_1) + \dots + a(i_k) - 1, a(i_1) + \dots + a(i_k) - 1 + o]$, where $\{i_1, \dots, i_k\}$ is the blowup face – this is the blowup face specified by Dido in type I, or empty in type II, plus all vertices which have been removed by intersection. The blowup of type I of a cell is a subdivision. A type II blowup reduces order ∞ to finite order. Tight quests (as created by tightification or intersection) remain always tight and have $o = 1$ and $a \equiv 0$. Order 0 remains order 0.

A quest and its descent have the same Ξ and b , and the dimension of the descent is one less.

A quest and its tightification have the same dimension. The complex of the tightification is a subcomplex. If a quest has a tightification which is not lost by blowup, then its order does not grow, and it drops if the tightification is won. A type II blowup reduces order ∞ to finite order.

A quest and its intersection with vertex j have same dimension and generating degree. The intersection is tight. The complex of the intersection is the subcomplex of all faces containing Ξ , but with j removed.

What is not determined by the restrictions. If Mephisto just provides in his move the complex of the main quest, the order of descent quests, and the generating degree and complex of tightification quests, then the remaining data are determined by the above restrictions (and Dido should figure out by herself).

Start. Initially, there is only the main quest, with Ξ consists only of the empty face. Its parameters d, b, o are arbitrary. Dido has the first move.

End. A quest is won by Dido if Ξ has no faces (not even the empty face). The game ends when she wins the main quest.