Let $K$ be a field of characteristic zero.

**Habitat.** A habitat is an equidimensional nonsingular variety $W$ together with a finite sequence of divisors $(E_1, \ldots, E_n)$, such that for each point $p \in W$, there is a system $u$ of regular parameters of the local ring of $W$ at $p$ such that for $i = 1, \ldots, n$, either $E_i$ does not pass through $p$ or there is a parameter in $u$ which serves as a local equation of $E_i$; in addition, we want that these local equations are pairwise distinct (or equivalently, the $E_i'$ are not allowed to have common components).

For any habitat, we define an operator $\Delta$ from the set of ideals on $W$ to itself, as follows. For $I \subset K[W]$, $\Delta(I)$ is the ideal generated by $I$ and all first order partial derivatives of elements in $I$.

**Straight subvariety.** A subvariety $Z$ of a habitat as above is called straight iff it is pure-dimensional, and for every point $p \in Z$, there is a system $u$ of regular parameters as above which additionally satisfies the condition that $Z$ is locally defined by a subset of $u$.

**Blowup.** Let $Z$ be a straight subvariety of a habitat $(W, (E_1, \ldots, E_n))$. The blowup along $Z$ is the habitat $(W', (E'_1, \ldots, E'_n, E_{n+1}))$, where $W'$ is the blowup of $W$ along $Z$, $E'_i$ is the strict transform of $E_i$ for $i = 1, \ldots, n$, and $E_{n+1}$ is the exceptional divisor.

**Singularity.** A singularity on a habitat as above is a finitely generated Rees algebra $A = \oplus_{i=0}^{\infty} A_i$ over $A_0 = K[W]$, i.e. sequence of ideals $A_i \subset K[W]$ such that $A_0 = K[W]$ and $A_i \cdot A_j \subseteq A_{i+j}$ and equality holds for sufficiently large indices $i, j$.

We say a singularity is of ideal-type iff there is a $b > 0$ and ideal $I$ such that $A_{nb} = I^n$ for all indices which are multiples of $b$, and $A_i = (0)$ otherwise. These singularities are denoted by $(I, b)$.

**Singular Locus.** The singular locus $\text{Sing}(A)$ of a singularity $A = \oplus_{i=0}^{\infty} A_i$ is the intersection of the zero sets of $\Delta_i^{i-1}(A_i)$, $i > 0$.

**Transform.** Let $Z$ be a straight subvariety in the singular set of $A$ as above. The transformed singularity on the blowup is $\oplus_{i=0}^{\infty} A'_i$, where $A'_i$ is such that $f^*(A_i) = \text{Ideal}(E_{n+1})^{i} \cdot A'_i$ for $i > 0$.

**Resolution.** A resolution of a singularity is a sequence of singularity-habitat pairs, where the next is the transform of the previous under blowup of a straight
subvariety in the singular locus, such that the last singularity has empty singular locus.

**Monomial Singularity.** A monomial singularity is an ideal-type singularity \((I, b)\) such that there exist integers \(a_1, \ldots, a_n\) such that \(I = \prod_{i=0}^n \text{Ideal}(E_i)^{a_i}\).

**Equivalence.** Two singularities \(A, B\) are equivalent – \(A \approx B\) – iff their singular loci coincide and, for any sequence of blowups in the center in the singular locus, the singular loci of the transforms of \(A\) and \(B\) coincide.

**Differential Closure.** A singularity \(A = \bigoplus_{i=0}^\infty A_i\) is closed iff \(\Delta(A_{i+1}) \subseteq A_i\) for all \(i > 0\).

The differential closure of a singularity \(A\) is the smallest closed singularity containing \(A\).

**Subhabitat.** A subhabitat of a habitat \((W, (E_1, \ldots, E_n))\) is a locally closed subvariety \(V \subset W\) intersecting each \(E_i\) transversally, together with the sequence of these transversal intersections.

**Restriction.** Let \(i : V \rightarrow W\) the inclusion map of a subhabitat \((V, \_\_\_\_\_)\) of \((W, \_\_\_\_)\).

Let \(i^* : K[W] \rightarrow K[V]\) be the corresponding homomorphism of function rings.

The restriction of a singularity \(B = \bigoplus_{i=0}^\infty B_i\) on \((W, \_\_\_)\) to \((V, \_\_\_)\) is defined as the singularity \(A = \bigoplus_{i=0}^\infty A_i\) where \(A_i := i^*(B_i)K[V]\).

If \(\text{Ideal}(V)^i\) is contained in \(B_i\) for all \(i > 0\), then we say that \(B\) restricts properly to \(V\). (In this case \(\text{Sing}(A) = \text{Sing}(B)\).)

**Extension.** Let \(W, V, i\) be as above. Let \(r : W \rightarrow V\) be a left inverse of \(i\).

The extension1 of a singularity \(A = \bigoplus_{i=0}^\infty A_i\) on \((V, \_\_\_)\) to \((W, \_\_\_)\) is defined as \(B = \bigoplus_{i=0}^\infty B_i\), where \(B_i := r^*(A_i) + \text{Ideal}(V)^i\).

Assume that \(A\) is closed. Then the extension2 is defined as the largest closed algebra which is contained in \(\bigoplus_{i=0}^\infty (r^*)^{-1}(A_i)\) (it may be constructed by induction on \(i\)).