CMI Summerschool 2012 / Course Schicho
Axioms for Gallimaufries

Basic. A gallimaufry $\mathcal{G}$ on a habitat $(W,(E_1,\dots,E_n))$ has a singular set $S \subset W$, a dimension $d \le \dim(W), d \ge 0$, a generating degree $b > 0$, an order $o \in \frac{1}{b}\mathbb{Z} \cup \{\infty\}$, and a monomial factor $a : [n] \to \frac{1}{b}\mathbb{Z}$.
The set S has dimension at most d.
Let $\{i_1,\dots,i_k\}$ be a subset of $[n]$. Let $X := E_{i_1} \cap \dots \cap E_{i_k}$. If $a(i_1)+\dots+a(i_k) \ge 1$, then $X \subset S$. If $a(i_1)+\dots+a(i_k)+o < 1$, then $X \cap S = \emptyset$.

Transform. Let $\mathcal{G}'$ be the transform of $\mathcal{G}$ along an admissible blowup. Then we have $S' \subseteq \pi^{-1}(S)$, $d' = d$, $b' = b$, and $a' : [n+1] \to \frac{1}{b}\mathbb{Z}$ extends a, and $a(n+1) \in [a(i_1)+\dots+a(i_k)-1, a(i_1)+\dots+a(i_k)-1+o]$ where $\{E_{i_1},\dots,E_{i_k}\}$ is the set of hypersurfaces containing the center.
Let $\mathcal{G}$ be a gallimaufry with order $o = \infty$. Then S contains an admissible center V_0 of dimension d. For the transform along the blowup of V_0, the order is finite.

Tight. A gallimaufry $\mathcal{G}$ is called tight iff $o = 1$ and $a(i) = 0$ for for all i sich that E_i intersects the singular locus. The transform of a tight gallimaufry is again tight.

Descent. Let $\mathcal{G}$ be a tight gallimaufry. The descent $\mathcal{G}\downarrow$, if exists, has the the following properties: $S\downarrow = S$, $d\downarrow = d-1$, $b\downarrow = b$.
If $E_i \cap S = \emptyset$ for $i = 1,\dots,n$, then the descent exists.
Let $\mathcal{G}$ be a tight gallimaufry with descent $\mathcal{G}\downarrow$. Let $\mathcal{G}'$ and $\mathcal{G}\downarrow'$ be the transforms along an admissible center. Then $\mathcal{G}\downarrow'$ is the descent of $\mathcal{G}'$.

Tightification. A gallimaufry $\mathcal{G}$ with $o > 0$ and $o < \infty$ has a tightfication $[\mathcal{G}]$. We have $[S] \subseteq S$, $[d] = d$, and $[\mathcal{G}]$ is tight.
Let $\mathcal{G}'$ and $[\mathcal{G}]'$ be the transforms on a center Z which is admissible for $[\mathcal{G}]$ (hence also for $\mathcal{G}$). Then we have $a'(n+1) = o + \sum_{i,Z\subset E_i} a_i - 1$ and $o' \le o$. Equality holds if and only if $[S]'$ is not resolved, and in this case $[S]'$ is the tightification of S'.

Intersection. Let $\mathcal{G}$ be a gallimaufry on a habitat $(W,(E_1,\dots,E_n))$. Let $j \le n$. Then the intersection gallimaufry $\mathcal{G}_{\cap j}$ is a tight gallimaufry on the habitat $(W,(E_1,\dots,E_{j-1},\emptyset,E_{j+1},\dots,E_n))$. We have $S_{\cap j} = S \cap E_i$, $d_{\cap j} = d$, $b_{\cap j} = b$.
Let $\mathcal{G}'$ and $(\mathcal{G}_{\cap j})'$ be the transforms along a center Z which is admissible for $\mathcal{G}_{\cap j}$. Then it is also admissible for $\mathcal{G}$, and $(\mathcal{G}_{\cap j})'$ is the intersection gallimaufry for $\mathcal{G}'$.

Monomial. A gallimaufry $\mathcal{G}$ with order $o = 0$ is called monomial. The transform of a monomial gallimaufry is again monomial.