Publication List

David E Speyer

1. "Tropical Linear Spaces", preprint `arXiv:math.CO/0410455`. To be incorporated as a chapter of my thesis.

   We define tropical analogues of the notions of linear space and Plücker coordinate and study their combinatorics. We introduce tropical analogues of intersection and dualization and define a tropical linear space built by repeated dualization and transverse intersection to be constructible. Our main result that all constructible tropical linear spaces have the same f-vector and are "series-parallel". We conjecture that this f-vector is maximal for all tropical linear spaces with equality precisely for the series-parallel tropical linear spaces. We present many partial results towards this conjecture.

   In addition we describe the relation of tropical linear spaces to linear spaces defined over power series fields and give many examples and counter-examples illustrating aspects of this relationship. We describe a family of particularly nice series-parallel linear spaces, which we term tree spaces, that realize the conjectured maximal f-vector and are constructed in a manner similar to the cyclic polytopes.

2. "A Broken Circuit Ring", with Nick Proudfoot, preprint `arXiv:math.CO/0410069` submitted to *Beitrage zur Geometrie und Algebra*

   Given a matroid M represented by a linear subspace $L \subset \mathbb{C}^n$ (equivalently by an arrangement of n hyperplanes in L), we define a graded ring $R(L)$ which degenerates to the Stanley-Reisner ring of the broken circuit complex for any choice of ordering of the ground set. In particular, $R(L)$ is Cohen-Macaulay, and may be used to compute the h-vector of the broken circuit complex of M. We give a geometric interpretation of Spec $R(L)$, as well as a stratification indexed by the flats of M.

3. "Tropical Mathematics", with Bernd Sturmfels, expansion of coauthor's Clay Public Lecture at Park City Mathematics Institute, preprint `arXiv:math.CO/0408099`

   These are the notes for the Clay Mathematics Institute Senior Scholar Lecture which was delivered by Bernd Sturmfels in Park City, Utah, on July 22, 2004. The topic of this lecture is the "tropical approach" in mathematics, which has gotten a lot of attention recently in combinatorics, algebraic geometry and related fields. It offers an an elementary introduction to this subject, touching upon Arithmetic, Polynomials, Curves, Phylogenetics and Linear Spaces. Each section ends with a suggestion for further research. The bibliography contains numerous references for further reading in this field.

4. "An Arctic Circle Theorem for Groves", with Kyle Petersen, presented at Formal Power Series and Algebraic Combinatorics 2004, *Journal of Combinatorial Theory: Series A*, to appear. preprint `arXiv:math.CO/0407171`

   In earlier work, Jockusch, Propp, and Shor proved a theorem describing the limiting shape of the boundary between the uniformly tiled corners of a random tiling of an Aztec diamond and the more unpredictable 'temperate zone' in the interior of the region. The so-called arctic circle theorem made precise a phenomenon observed in random tilings of large Aztec diamonds. Here we examine a related combinatorial model called groves. Created by Carroll and Speyer as combinatorial interpretations for Laurent polynomials given by the cube recurrence, groves have observable frozen regions which we describe precisely via asymptotic analysis of a generating function. Our approach also provides another way to prove the arctic circle theorem for Aztec diamonds.

5. "The Tropical Totally Positive Grassmannian", with Lauren Williams *Journal of Algebraic Combinatorics*, to appear preprint `arXiv:math.CO/0312297`

   Tropical algebraic geometry is the geometry of the tropical semiring $(\mathbb{R}, \min, +)$. The theory of total positivity is a natural generalization of the study of matrices with all minors positive. In this paper we introduce the totally positive part of the tropicalization of an arbitrary affine variety, an object which has the structure of a polyhedral fan. We then investigate the case of the Grassmannian, denoting the resulting fan $\mathrm{Trop}^+ Gr_{k,n}$. We show that $\mathrm{Trop}^+ Gr_{2,n}$ is the Stanley-Pitman fan, which is combinatorially the fan dual to the (type A_{n-3}) associahedron, and that $\mathrm{Trop}^+ Gr_{3,6}$ and $\mathrm{Trop}^+ Gr_{3,7}$ are closely related to the fans dual to the types D_4 and E_6 associahedra. These results are reminiscent of the results of Fomin and Zelevinsky, and Scott, who showed that the Grassmannian has a natural cluster algebra structure which is of types A_{n-3}, D_4, and E_6 for $Gr_{2,n}$, $Gr_{3,6}$, and $Gr_{3,7}$. We suggest a general conjecture about the positive part of the tropicalization of a cluster algebra.

6. "Horn's Problem, Vinnikov Curves and the Hive Cone", *Duke Mathematical Journal*, to appear. preprint `arXiv:math.AG/0311428`

   A Vinnikov curve is a projective plane curve which can be written in the form $\det(xX + yY + zZ) = 0$ for X, Y and Z positive definite Hermitian $n \times n$ matrices. Given three n-tuples of positive real numbers, α, β and γ, there exist A, B and $C \in \mathrm{GL}_n\mathbb{C}$ with singular values α, β and γ and $ABC = 1$ if and only if there is a Vinnikov curve passing through the $3n$ points $(-1 : \alpha_i^2 : 0)$, $(0 : -1 : \beta_i^2)$ and $(\gamma_i^2 : 0 : -1)$. Knutson and Tao proved that another equivalent condition for such A, B and C to exist is that there is a hive whose boundary is $(\log\alpha, \log\beta, \log\gamma)$. The logarithms of the coefficients of F approximately form such a hive; this leads to a new proof of Knutson and Tao's result. This paper uses no representation theory and essentially no symplectic geometry. In their place, it uses Viro's patchworking method and a topological description of Vinnikov curves.

7. "Reconstructing Trees from Subtree Weights" with Lior Pachter *Applied Mathematics Letters* **17** (2004), p. 615 - 621.

    The tree-metric theorem provides a necessary and sufficient condition for a dissimilarity matrix to be a tree metric, and has served as the foundation for numerous distance-based reconstruction methods in phylogenetics. Our main result is an extension of the tree-metric theorem to more general dissimilarity maps. In particular, we show that a tree with n leaves is reconstructible from the weights of the m-leaf subtrees provided that $n \geq 2m - 1$.

8. "The Tropical Grassmannian" with Bernd Sturmfels *Advances in Geometry*, to appear. Volume 4, Issue 3 (2004). Pages 389 - 411

    In tropical algebraic geometry, the solution sets of polynomial equations are piecewise-linear. We introduce the tropical variety of a polynomial ideal, and we identify it with a polyhedral subcomplex of the Gröbner fan. The tropical Grassmannian arises in this manner from the ideal of quadratic Plücker relations. It parameterizes all tropical linear spaces. Lines in tropical projective space are trees, and their tropical Grassmannian $\mathcal{G}_{2,n}$ equals the space of phylogenetic trees studied by Billera, Holmes and Vogtmann. Higher Grassmannians offer a natural generalization of the space of trees. Their faces correspond to monomial-free initial ideals of the Plücker ideal. The tropical Grassmannian $\mathcal{G}_{3,6}$ is a simplicial complex glued from 1035 tetrahedra.

9. "The Cube Recurrence" with Gabriel Carroll *Elec. Jour. of Comb.* Vol. 11(1) (2004) #R72

    We construct a combinatorial model that is described by the cube recurrence, a nonlinear recurrence relation introduced by Propp, which generates families of Laurent polynomials indexed by points in $\mathbb{Z}^3$. The elements of our model are forests with certain connectivity conditions that we term "groves". In the process, we prove several conjectures of Propp and of Fomin and Zelevinsky, and we obtain a combinatorial interpretation for the terms of Gale-Robinson sequences. We also indicate how the model might be used to obtain some interesting results about perfect matchings of certain bipartite planar graphs.

10. "Perfect Matchings and the Octahedron Recurrence" preprint `arXiv:math.CO/0402452`

    We study a recurrence defined on a three dimensional lattice and prove that its values are Laurent polynomials in the initial conditions with all coefficients equal to one. This recurrence was studied by Propp and by Fomin and Zelivinsky. Fomin and Zelivinsky were able to prove Laurentness and conjectured that the coefficients were 1. Our proof establishes a bijection between the terms of the Laurent polynomial and the perfect matchings of certain graphs, generalizing the theory of Aztec diamonds. In particular, this shows that the coefficients of this polynomial, and polynomials obtained by specializing its variables, are positive, a conjecture of Fomin and Zelevinsky.

11. "Every Tree is 3-Equitable" with Zsuzsanna Szansiszlo *Discrete Mathematics* **220**, (2000) 283-289

    Let G be a graph whose vertices are labeled with the integers 0, 1, $\ldots i$. Label each edge with the absolute value of the difference between its endpoints. A labeling is called equitable if, for any two numbers a and b in $\{0, \ldots, i\}$, the number of vertices with label a differs by at most one from the number with label b and the same property holds for the number of edges with each label. It is conjectured that every tree has an equitable labeling for every i. We prove this conjecture for $i = 2$.