Research description

My current research interest is the calculation of the L-functions of Shimura varieties, or more precisely of the L-functions of intersection complexes of the Baily-Borel compactification of Shimura varieties.

The Shimura varieties associated to the group $\mathbf{GL}_2$, i.e. the modular curves, have been intensively studied by many authors. If Y is a modular curve, its Baily-Borel compactification $j : Y \longrightarrow Y^*$ is obtained by adding a finite number of points, called cusps. As the compactification Y^* is smooth, the intersection complex with coefficients in a local system $\mathcal{F}$ on Y is simply $j_*\mathcal{F}$. In this case, it is possible to calculate the L-function of this intersection complex, and this function turns out to be an alternate product of L-functions of modular forms and of zeta-functions (as was shown by, among others, Eichler, Shimura, Deligne and Ihara).

The L-function is a product of local factors L_p, where p is a prime number, and these local factors are the functions that have been calculated for modular curves. The essential case is when both Y^* and $j_*\mathcal{F}$ have good reduction at p. Then the local factors L_p depends only on the reduction mod p of Y^* and $j_*\mathcal{F}$.

In the case of modular curves, there are two methods available to calculate L_p : the congruence method and the comparison of the Grothendieck-Lefschetz fixed point formula and the Arthur-Selberg trace formula. Only the second method seems to have a chance to work in higher dimensions. It was applied successfully to the Shimura varieties associated to the groups $R_{E/\mathbb{Q}}\mathbf{GL}_2$, where E is a totally real number field, and $\mathbf{GU}(2,1)$ (this was done by Brylinski and Labesse for the first case, and by various people in the book "The zeta-functions of Picard modular surfaces" in the second case).

To apply this method to a compact Shimura variety (associated to a group $\mathbf{G}$), one has to calculate the trace of a Hecke operator times a power of the Frobenius endormophism on the cohomology of a local system associated to a representation of the group $\mathbf{G}$; this was done by Kottwitz in the case of PEL Shimura varieties. Apart from this problem, the two major difficulties are the stabilization of the trace formula and the fundamental lemma.

If the Shimura variety is not compact, a third difficulty is the contribution of the points at infinity. It is expected that the result will be easier to compare to the geometric side of the trace formula if one uses the intersection cohomology of the Baily-Borel compactification (or, more generally, the cohomology of the intersection complex with coefficients in a local system associated to a representation of the group). So the first step is to calculate the trace of a Hecke correpondance times a power of the Frobenius endomorphism on this intersection cohomology.

This calculation was done in my thesis for the Shimura varieties associated to unitary groups over $\mathbb{Q}$ (of arbitrary rank) and for a trivial Hecke correspondance. Using the same method, I then treated the case of Siegel modular varieties and nontrivial Hecke correspondances. I expect the method to work for any Shimura varieties (if the strata of the Baily-Borel compactification are PEL), but haven't checked this.

Now it would be nice to finish the calculation of the local factor L_p, at least for the above-mentionned Shimura varieties. Recent progress has been made on the fundamental lemma :

Laumon and Ngo have proved the (endoscopic) fundamental lemma for unitary groups over function fields, and then Waldspurger showed that this implies the fundamental lemma for unitary groups over number fields (at almost all places).

So the main remaining difficulty, in the case of unitary groups, is the stabilization of the trace formula. The term coming from the Shimura variety itself was stabilized by Kottwitz. I will attempt to stabilize the remaining terms and to deduce that the L-function is indeed of the expected form.