Research Description: Mohammed Abouzaid

I will continue working on ideas motivated by homological mirror symmetry and by the nearby Lagrangian conjecture. I will focus on the following problems:

1) Understanding homological mirror symmetry for Calabi-Yau manifolds from the point of view of tropical geometry: On the symplectic side, one can produce a rich collection of Lagrangians using tropical geometry, so the main technical point is to compute Floer homology by tropical techniques. I am working on generalizing preliminary results for the mirrors of lines bundles. On the complex side, work of Gross-Siebert provides a potential way to understand the category of coherent sheaves on a complex manifold by studying singular affine spaces associated to toric degenerations. I plan to study the connection between tropical and singular affine manifolds in order to prove that the Fukaya category of tropical Lagrangians is equivalent to the mirror category of coherent sheaves. Keeping in mind the example of the quintic 3-fold, I will begin with the case of projective hypersurfaces.

2) Applying methods of surgery theory to the nearby Lagrangian conjecture: Ideas of Nadler and Fukaya, Seidel and Smith on the Fukaya category are expected to imply that every exact Lagrangian $L \subset T^*M$ is homotopy equivalent to M. To attack the diffeomorphism problem, the surgery program suggests considering first the intersection pairing at the cochain level. This yields a class in the L-theory of the fundamental group which is an obstruction to handle-sliding and hence makes sense in the setting of CW complexes satisfying Poincaré duality. The theory of Lagrangian skeleta developed by Biran and Eliashberg is particularly well suited to this problem (the zero section of the cotangent bundle is the example of interest). I will study the space of such Lagrangian skeleta with a view toward implementing this half of the surgery program. The other half concerns tangential data. I plan to attack the problem by using higher dimensional moduli spaces of holomorphic curves.

3) Recovering the Fukaya category of symplectic manifolds from tropical decompositions: Recent work of Fukaya-Seidel-Smith and Nadler-Zaslow has given a rather complete picture of the Fukaya category of cotangent bundles. Tropical geometry provides a decomposition of many symplectic manifolds into elementary parts which are products of cotangent bundles of tori and "higher dimensional pairs of pants." There is preliminary evidence that this decomposition induces good restriction functors at the level of derived Fukaya categories, analogous to Viterbo's functoriality result for symplectic cohomology. In joint

work with Paul Seidel, I plan to work on using these various functors to construct the Fukaya categories of such symplectic manifolds from a Čech-like construction in the setting of A_∞ categories.